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SOME PROBLEMS CONCERNING THE THEORY OF PULSED NEUTRON EXPERIMENTS

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A b s t r a c t

Some problems of the theoretical interpretation of pulsed neutron experiments are investigated on the basis of the energy-dependent Boltzmann equation. The relation of the infinite medium theory to the finite medium experiments is discussed in detail. A calculation is performed in P_1L_1 approximation in order to determine the shape of neutron flux and the extrapolation length. It is shown that the existence of an asymptotic region is not required for applying the infinite medium theory to the finite medium measurements.

SOME PROBLEMS CONCERNING THE THEORY OF PULSED NEUTRON EXPERIMENTS

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1. Introduction

The technique of pulsed neutron experiments has been developed already before the First Geneva Conference on the basis of the work performed by Dardel [1] , Antonov et. al. [2] , and others. The simplest theory of pulsed neutron measurements is based on the solution of the one-group diffusion equation. Owing to the increasingly sophicticated measuring techniques and to the phenomenon of diffusion cooling, it has become necessary to work out the energy dependent treatment and the transport theory approach.

After a neutron pulse, injected into a moderator sample, the flux has the form:

$$\phi(r,E,t)=\sum_{n=0}^{\infty} a_n \phi_{\lambda_n}(r,E,t)e^{-\lambda_n t} \qquad /1.1/$$

where λ_n are the eigenvalues of the following equation: [3a]

$$\left[-\frac{\lambda}{v}+\Omega\,\mathrm{grad}+\sum_s(E)+\sum_a(E)\right]\phi_\lambda(r,E,\Omega)=$$
$$=\int_0^\infty dE'\int_{4\pi} d\Omega'\sum_s(E,E',\Omega,\Omega')\,\phi_\lambda(r,E',\Omega') \qquad /1.2/$$

and $\lambda_0 < \lambda_1 < \ldots \lambda_n$

For large t:

$$\phi(r,E,\Omega,t)\approx a_{\lambda_0}\phi_{\lambda_0}(r,E,\Omega)e^{-\lambda_0 t} \qquad /1.3/$$

Thus measuring the decay of the neutron pulse, the lowest eigenvalue can be obtained.

For the interpretation of the measurements the formula

$$\lambda_0=\alpha_0+DB^2-CB^4 \qquad /1.4/$$

is used, where B^2 is the material buckling taken to be equal to the geometric one. α_0D and C can be determined from measurements at various geometric bucklings. Consequently, the following questions arise:

1. What is the physical meaning of α_0D and C and what is their relation to the cross-sections.
2. How the geometric buckling can be determined from the actual dimension of a moderator sample.

So far, the first problem has been the main subject of theoretical considerations. The simplest way leading to /1.4/ has been the introduction of the diffusion cooling term into the formula [1]

$$\lambda=\sum_a v+D(B^2)B^2$$

In that way, a term in B^4 can be found.

Nelkin has obtained coefficients of /1.4/ by a variational approach, using the "neutron temperature" as a variational parameter.

Supposing space-energy separability of $\phi(x,E,\mu)$, a one-velocity equation can be derived. /3 b/ From this one-velocity equation Sjöstrand [5] has deduced an exact expression for $\lambda(B)$ and its /1.4/ expansion.

Singwi [6] ,Purohit [7] and others [8] ,applying P_1 approximation, have determined the coefficients α_0 D and C with the help of Laguerre expansion. They kept onl- the first two terms of the Laguerre expansion i.e. L_1 approximation was used, but the correction indicated by higher order Laguerre polynomials was also calculated.

For the case of infinite medium, Nelkin [9] has derived $\lambda(B)$ from the energy dependent Boltzmann equation with the help of Fourier transform. In the following we shall deal with the relation of this infinite medium theory to the finite sample measurements.

2. Application of Nelkin's infinite medium theory to finite sample measurements

In the following our considerations are restricted to the one dimensional case and to the use of an isotropic scattering kernel, thus the Eq./1.2/ becomes

$$\left[-\frac{\lambda}{v}+\mu\frac{\partial}{\partial x}+\Sigma_s(E)+\Sigma_a(E)\right]\phi_\lambda(E,x,\mu)=$$
$$=\frac{1}{2}\int_0^\infty dE'\Sigma_s(E'E)\int_{-1}^{1}\phi_\lambda(E',x,\mu')d\mu' \qquad /2.1/$$

From now on we consider λ as a given quantity, usually as the lowest eigenvalue of Eq. /1.2/

We try to solve /2.1/ for

$$\phi_\lambda(E,x,\mu)=F_\lambda(E,\mu,B)e^{iBx} \qquad /2.2/$$

Substituting /2.2/ into /2.1/ we have

$$\left[-\frac{\lambda}{v}+\mu iB+\Sigma_s(E)+\Sigma_a(E)\right]F_\lambda(E,\mu,B)=$$
$$=\frac{1}{2}\int_0^\infty dE'\Sigma_s(E'E)\int_{-1}^{1}F_\lambda(E'\mu B)d\mu' \qquad /2.3/$$

This is the same equation which Nelkin has obtained by Fourier transform [9]. But the above formulation has an advantage over that of Nelkin's. Namely, Fourier transform usually implies the boundary condition too. Here, however, finding the set of eigenvalues /2.3/ :

$B_0, B_1 \ldots\ldots\ldots\ldots B_k \ldots\ldots$

one can write down the total solution of Eq./2.1/ which satisfies the boundary condition of a finite sample. [10] This procedure will be carried out in P_1L_1 approximation in the next paragraphs.

Now, however, for the sake of further discussion, we write down the flux for the lowest λ_0 eigenvalue:

$$\phi_0(x,E,t) = e^{-\lambda_0 t}\left[F_0(E,B_0)\cos B_0 x + \sum_{k=0}^{\infty} F_0(E,B_k)e^{iB_k x}\right] \qquad /2.4/$$

where

$$\phi_0(x,E,t) = \int_{-1}^{1} \phi(x,E,\mu,t)\,d\mu$$

$$F_0(E,B) = \int_{-1}^{1} F(E,\mu,B)\,d\mu$$

The index λ_0 is omitted.

The first term in /2.4/ is written separately, since it represents the asymptotic part of the flux. In fact, if all B-s, except the first, are imaginary /we shall see later when this is the case/, then far from boundaries, the separately written first term will predominate:

$$\phi(x,E,t) \approx e^{-\lambda_0 t} F_0(E,B_0)\cos B_0 x$$

The extrapolation length is the distance from the actual boundary, where this asymptotic flux vanishes, i.e.:

$$\cos B_0(a+d) = 0$$

Hence

$$B_0 = \frac{\pi}{a+d} \qquad /2.5/$$

Thus B_0 can be obtained, if d is known. d can be determined from the boundary condition of the transport theory./E.g. from Marshak's boundary conditions/. We shall deal with this question in 4.

If more than one pair of B_k-s are real, then there is no asymptotic region, thus extrapolation length, in the usual sense, does not exist. However this case does not cause any difficulty in principle, because a value of d where the first term in /2.4/ vanishes, even now exists, consequently, a geometric buckling B can be assigned to the finite sample. Accordingly, the existence of an asymptotic region is not required for the application of the infinite medium theory to the finite sample experiments. However, if an asymptotic region exists, the interpretation of the measurements is simpler, because the extrapolation length is easier to determine. Therefore it is of interest to know a criterion of the existence of an asymptotic region.

3. The set of B_k values at given λ_0. The criterion of the existence of an asymptotic region

We are going to consider the set of B_k values in P_1L_1 approximation, if the lowest decay constant λ_0 is given. In this approximation there are two pairs of roots.+ /Generally, in P_NL_J approximation there are pairs of roots./

We start from the P_1 equations with the following scattering kernel:

$$\Sigma_s(E,E',\Omega,\Omega) = \frac{1}{4\pi}\left[\Sigma_s(E\to E') + 3P_1(\Omega\Omega')\bar{\mu}(E)\Sigma_s(E)\delta(E-E')\right] \qquad /3.1/$$

and the notation

$$\Sigma_{tr}(E) = \Sigma_s(E)(1-\bar{\mu}(E))$$

is introduced. For the sake of simplicity, we deal only with moderators obeying 1/v absorption law. We note briefly: $\frac{\lambda}{v} - \Sigma_a = \frac{\chi}{v}$ Since we discuss a non-multiplying medium: $\chi > 0$. Thus the one-dimensional equations in P_1 approximation are the following:

$$\frac{1}{3(\Sigma_{tr}(E)-\frac{\chi}{v})}\frac{\partial^2}{\partial x^2}\Phi_0(E,x) + \frac{\chi}{v}\Phi_0(E,x) = \qquad /3.2/$$

$$= \int_0^\infty \Sigma_s(E'\to E)\Phi_0(E,x)dE' - \Sigma_s(E)\Phi_0(E,x)$$

$$\Phi_1(E,x) = -\frac{1}{3(\Sigma_{tr}(E)-\frac{\chi}{v})}\frac{\partial}{\partial x}\Phi_0(E,x) \qquad /3.3/$$

Let us expand $\Phi_0(E,x)$ and $\Phi_1(E,x)$ in the generalized Laguerre polynomials of the order of unity:

$$\Phi_n(E,x) = M(E)\sum_{k=0}^{\infty}\Phi_n^k(x)L_k^{(1)}(E) \qquad n=0,\ 1 \qquad /3.4/$$

Substituting /3.4/ into /3.2/ and /3.3/, multiplying the resulting equations by $L_i^{(1)}(E)$ and integrating over E, we get:

+ The set of B_k values are called roots, since they are roots of a characteristic equation.

$$\sum_{k=0}^{\infty}\left[\frac{t_{ik}}{3}\frac{\partial^2}{\partial x^2}+\gamma_{ik}+\varkappa\left(\frac{1}{v}\right)_{ik}\right]\phi_0^k(x)=0 \qquad /3.5/$$

$$\phi_1^i(x)=-\frac{\partial}{\partial x}\sum_{k=0}^{\infty}t_{ik}\phi_0^k(x) \qquad /3.6/$$

where

$$\phi_n^k(x)=\frac{1}{k+1}\int_0^{\infty}\phi_n(E,x)L_k^{(1)}(E)dE \qquad n=0,1 \qquad /3.7a/$$

$$t_{ik}=\int_0^{\infty}\frac{1}{\Sigma_{tr}(E)-\frac{\varkappa}{v}}M(E)L_i^{(1)}(E)L_k^{(1)}(E)dE \qquad /3.7b/$$

$$\left(\frac{1}{v}\right)_{ik}=\int_0^{\infty}\frac{1}{v}M(E)L_i^{(1)}(E)L_k^{(1)}(E)dE \qquad /3.7c/$$

$$\gamma_{ik}=-\frac{1}{2}\int_0^{\infty}dE\int_0^{\infty}dE'\Sigma_s(E'\to E)M(E')\left[L_k^{(1)}(E')-L_k^{(1)}(E)\right]\left[L_i^{(1)}(E')-L_i^{(1)}(E)\right] \qquad /3.7d/$$

For transformation of γ_{ik} we have used the condition of detailed balance:

$$M(E)\Sigma_s(E\to E')=M(E')\Sigma_s(E'\to E)$$

It is evident from /3.7d/ that $\gamma_{i0}=0$ and $\gamma_{ii}<0$ if $i>0$

As we have said, we want to find the solution in the form of

$$\phi_n^k(x)=F_n^k(x)e^{iBx} \qquad n=0,1 \qquad /2.2'/$$

We shall use L_1 approximation i.e. we shall keep only terms corresponding to i=0, 1 and k=0, 1; then we have:

$$\left[-\frac{B^2 t_{00}}{3} + \varkappa\left(\frac{1}{v}\right)_{00}\right] F_0^0(B) + \left(-\frac{B\, t_{01}}{3} + \varkappa\left(\frac{1}{v}\right)_{00}\right) F_0^1(B) = 0$$

$$\left[-\frac{B^2 t_{01}}{3} + \varkappa\left(\frac{1}{v}\right)_{01}\right] F_0^0(B) + \left(-\frac{B^2 t_{11}}{3} + \varkappa\left(\frac{1}{v}\right)_{11} - |\gamma_{11}|\right) F_0^1(B) = 0 \qquad /3.5'/$$

$$F_1^0(B) = -\frac{iB}{3}\left[t_{00} F_0^0(B) + t_{01} F_0^1(B)\right]$$

$$F_1^1(B) = -\frac{iB}{3}\left[t_{01} F_0^0(B) + t_{11} F_0^1(B)\right] \qquad /3.6'/$$

The Eqs. /3.4'/ are a set of homogeneous equations, and the roots of its characteristic equation are the possible values of B.We find that if

$$\frac{1{,}5\, \varkappa\left(\frac{1}{v}\right)_{00} - |\gamma_{11}|}{t_{00}\, t_{11} - t_{01}^2} < 0 \qquad /3.8/$$

then one pair of roots is real, the other is imaginary i.e.:

$$B_0 = \pm B \qquad B_1 = \pm i\nu$$

This means that if the condition /3.8/ is fulfilled, there is an asymptotic region in the moderator sample. Usually $t_{00}\, t_{11} - t_{01} > 0$ therefore the asymptotic region always exists, unless $\varkappa$ is too large.

Thus if /3.8/ is fulfilled, the flux is:

$$\Phi_0(E,x) = A_B F_0(E,B) \cos Bx + A_\nu F_0(E,\nu) \operatorname{ch} \nu x \qquad /3.9/$$

if not

$$\Phi_0(E,x) = A_0 F_0(E,B) \cos Bx + A_1 F_0(E,B_1) \cos B_1 x \qquad /3.9'/$$

for a slab. The fulfilment of /3.8/ depends on the magnitude of $|\gamma_{11}|$. This is physically understandable since this magnitude characterizes the "thermalization power" of a moderator.

For illustration we apply the above approach to water.

We suppose that $\sum_{tr}(E) \sim E^{-\frac{1}{2}}$ .The γ_{11} can be calculated from the measured value of C from the formula [7] [8] :

$$C = \frac{\sqrt{\pi}\, D^2}{2 v_0 |\gamma_{11}|}$$

We use $C=0,036\ cm^4.sec^{-1}$, and get $|\gamma_{11}|=1,39\ cm^{-1}$.

It follows from/3,8/ that if $\chi < 1,6.10^5\ sec^{-1}$, then there is an asymptotic region. The time taken for measurements is only up to $0,26.10^5 sec^{-1}$. Owing to the good thermalizing properties of water./This is probably not the case for moderators with bad thermalizing properties./

One can visualize the asymptotic region in an interesting way with the help of /3.9/ let us write: [11]

$$\frac{\Delta \int_0^\infty \phi_0(E,x)\,dE}{\int_0^\infty \phi_0(E,x)\,dE} = -B^2_{eff} \qquad /3.10/$$

If we disregard the second term in /3.9/ i.e. take into account only the asymptotic flux, then $B=B_{eff}$. Thus the spatial variation of B^2_{eff} and its departure from B_2 indicate the departure from the asymptotic region. In Fig.1. B^2_{eff} for a slab of water can be seen. Here $\chi = 0,8.10^4 sec^{-1}$, $B^2=0,296\ cm^{-2}$. This corresponds to a slab of 2.55 cm half-thickness. The solid curve represents B^2_{eff} calculated from /3.9/ .Gelbard et.al.have also calculated B^2_{eff} in P_3 approximation using electronic computer.[11] The dashed curve shows their result. The agreement between the two results is only qualitative, but presumably the difference is mainly due to the large error of P_1 approximation at the boundary of a slab.

4. The buckling dependence of the extrapolation length

Let us deal with the case when the flux has the form /3.9/ , i.e. an asymptotic region exists. We have yet to impose the boundary condition on the solution /3.9/ .In the case of infinite medium, the flux must have a finite value at infinity. Therefore A =0. Since a physically reasonable solution is everywhere positive, thus B=0. For a slab of half-thickness a one can use Marshak's boundary condition. In P_1 approximation it has the form:

$$-\frac{1}{2}\phi_0(a,E)+\phi_1(a,E)=0$$

This equation in L_1 approximation becomes:

$$-\frac{1}{2}\phi_0^0(a)+\phi_1^0(a) = 0$$

$$-\frac{1}{2}\phi_0^1(a)+\phi_1^1(a) = 0 \qquad /4.1/$$

From Eqs. /3.3/ in L_1 approximation we have

$$\Phi_1^0(a)=-\frac{1}{3}\frac{\partial}{\partial x}\left[t_{00}\Phi_0^0(x)+t_{01}\Phi_0^1(x)\right]_{x=a}$$

$$\Phi_1^1(a)=-\frac{1}{3}\frac{\partial}{\partial x}\left[t_{01}\Phi_0^0(x)+t_{11}\Phi_0^1(x)\right]_{x=a} \qquad /4.2/$$

Substituting /4.2/ into /4.1/ we get the following set of equations:

$$A_B\left[F_0^0(B)\cos Ba-\left(t_{00}F_0^0(B)+t_{01}F_0^1(B)\right)\frac{2B}{3}\sin Ba\right]+$$

$$+A_\nu\left[F_0^0(\nu)\operatorname{ch}\nu a+\left(t_{00}F_0^0(\nu)+F_0^1(\nu)t_{01}\right)\frac{2\nu}{3}\operatorname{sh}\nu a\right]=0$$

$$A_B\left[F_0^1(B)\cos Ba-\left(t_{01}F_0^0(B)+t_{11}F_0^1(B)\right)\frac{2B}{3}\sin Ba\right]+$$

$$+A_\nu\left[F_0^1(\nu)\operatorname{ch}\nu a+\left(t_{01}F_0^0(\nu)+t_{11}F_0^1(\nu)\right)\frac{2\nu}{3}\operatorname{sh}\nu a\right]=0$$

/4.3/

From the characteristic equation of /4.3/ the extrapolation length d, where the asymptotic part of /3.9/ vanishes, can be obtained as:

$$\cos B(a+d)=0$$

Let us take $F_0^0(B)=F_0^0(\nu)=1$ then:

$$d=\frac{1}{B}\operatorname{arctg}\left[\frac{2B}{3}\left(t_{00}+F_0^1(B)t_{01}\right)\frac{1-\dfrac{t_{11}F_0^1(B)+t_{01}}{t_{01}F_0^1(B)+t_{00}}\cdot\dfrac{P}{q F_0^1(\nu)}}{1-\dfrac{F_0^1(B)}{F_0^1(\nu)}\cdot\dfrac{P}{q}}\right]= \qquad /4.4/$$

$$\frac{2}{3}\left(t_{00}+F_0^1(B)t_{01}\right)\frac{1-\dfrac{t_{11}F_0^1(B)+t_{01}}{t_{01}F_0^1(B)+t_{00}}\cdot\dfrac{P}{q F_0^1(\nu)}}{1-\dfrac{F_0^1(B)P}{F_0^1(\nu)q}}-\frac{8}{81}B^2\left(t_{00}+F_0^1(B)t_{01}\right)^3\left(\frac{1-\dfrac{t_{11}F_0^1(B)+t_{01}}{t_{01}F_0^1(B)+t_{00}}\cdot\dfrac{P}{q F_0^1(\nu)}}{1-\dfrac{F_0^1(B)P}{F_0^1(\nu)q}}\right)^3\cdots$$

ere

$$P=1+\left(t_{00}+F_0^1(\nu)t_{01}\right)\frac{2\nu}{3}\operatorname{th}\nu a \qquad /4.5/$$

$$q=1+\left(t_{11}+\frac{t_{01}}{F_0^1(\nu)}\right)\frac{2\nu}{3}\operatorname{th}\nu a$$

From Eqs. /3.5'/:

$$F_0^1(B) = -\frac{\frac{B^2}{3}t_{01} - \mathcal{X}\left(\frac{1}{v}\right)_{01}}{\frac{B^2}{3}t_{11} + |\gamma_{11}| - \mathcal{X}\left(\frac{1}{v}\right)_{11}} \qquad /4.6a/$$

$$F_0^1(\nu) = -\frac{\frac{\nu^2}{3}t_{01} + \mathcal{X}\left(\frac{1}{v}\right)_{00}}{\frac{\nu^2}{3}t_{10} + \mathcal{X}\left(\frac{1}{v}\right)_{01}} \qquad /4.6b/$$

The factor $t_{00}+F_0^1/B/t_{01}$ has a simple physical meaning. Let us put down separately the asymptotic term from /3.9/:

$$\phi_{as}(E,x) = A_B M(E)\left[L_0^{(1)}(E) + F_0^1(B)L_1^{(1)}(E)\right]\cos Bx \qquad /4.7/$$

Averaging $\left[\sum_{tr}(E) - \frac{\mathcal{X}}{v}\right]^{-1} = \lambda_{tr}(E)$ over /4.7/

$$\frac{\int_0^\infty dE\, \lambda_{tr}(E)\phi_{as}(E,x)}{\int_0^\infty dE\, \phi_{as}(E,x)} = t_{00} + F_0^1(B)t_{01} \quad \langle \lambda_{tr}(E)\rangle_{\mathcal{X}}$$

Thus the above factor gives the buckling dependence of the extrapolation length resulting from the diffusion cooling, taking into account the increase in transport mean free path, caused by $\mathcal{X}$

The last term in /4.4/ generally will not be, 1, not even in the case $\mathcal{X}=0$, unless $t_{01}=0$./E.g if $\sum_{tr}$=const./ The case $\mathcal{X}=0$ corresponds to Milne's problem with no absorption, consequently it gives the same d we should get by solving the infinite half-space problem. It is known that the exact result in the constant cross-section approximation is

$$d = 0{,}7104 \langle \lambda_{tr}(E)\rangle_0 \qquad (t_{00} = \langle \lambda_{tr}(E)\rangle_0)$$

If the transport cross-section is energy dependent, the extrapolation length is differing from 0,7104. For that case Nelkin uses a variational approach that leads to [12]

$$d = \frac{3}{8}\frac{\langle \lambda_{tr}^2(E)\rangle_0}{\langle \lambda_{tr}(E)\rangle_0} + \frac{1}{3}\langle \lambda_{tr}(E)\rangle_0 \qquad /4.8/$$

The averages have to be taken in terms of the Maxwellian flux at temperature T of the infinite half-space. Applying /4.8/ to water we find

$$d=0,758 \langle \lambda_{tr}(E) \rangle_0 \qquad /4.9/$$

i.e. the extrapolation length increases because of the energy dependence of the transport mean free path.

We also get an increase from the formula /4.4/ In fact, applying /4.4/ to water, for $x=0$, we get:

$$d=0,738 \langle \lambda_{tr}(E) \rangle_0 \qquad (t_{00}=\langle \lambda_{tr}(E) \rangle_0) \qquad /4.10/$$

as compared to $d=2/3 \ \langle \lambda_{tr}(E) \rangle_0$ which is given by P_1 in constant cross-section approximation.

Let us deal with the case $x \neq 0$. Gelbard et. al. [11] have made a numerical approach for finding the extrapolation length for slabs. Considering an infinite slab and assuming that

$$\phi(t,z,E)=e^{-\lambda_0 t}\phi(z,E)$$

λ_0 as well as $\phi(z,E)$ have been determined for a given thickness 2a. They worked in P_3 approximation, applying Marshak's boundary condition. Thus, obtaining λ_0 and using the relation /1.4/ B^2 was found. From B^2 the value of d was obtained by /2.5/ .However, we consider the buckling dependence of d/d_0 where d_0 is the extrapolation length corresponding to B=0. In Fig.2. the buckling dependence from /4.4/ and Gelbard's result are shown by the solid and the dashed curve, respectively. There is no great difference between the two results. The departure of /4.9/ from/4.10/ is due to the P_1 approximation. /Gelbard et. al also have obtained 0,758/. This means that the P_1L_1 approximation can be applied to the calculation of the extrapolation length if we correct the value 0,738 to 0,758.

It is interesting to plot the quantity $\frac{d}{d_0}\frac{\langle \lambda_{tr}(E) \rangle_0}{\langle \lambda_{tr}(E) \rangle_x}$. It shows the buckling dependence without diffusion cooling.[+] This buckling dependence has no clear physical meaning. In Fig.2. the dotted curve represents the $\frac{d}{d_0}\frac{\langle \lambda_{tr}(E) \rangle_0}{\langle \lambda_{tr}(E) \rangle_x}$ ratio calculated by Gelbard et.al., the dotted-line is the same quantity obtained by /4.4/ .

Above calculations can be carried out, even if two pairs of roots are real only instead of $\mathrm{ch}\,\nu a$, $\nu\,\mathrm{sh}\,\nu a$, $\nu\,\mathrm{th}\,\nu a$ we must write $\cos B_1 a, -B_1 \sin B_1 a, -B_1 \mathrm{tg} B a$ respectively. We get a d value even now, but it has no physical meaning like the extrapolation length has, because there is no asymptotic region. But this d can also be used for calculating the geometric buckling, in the same way as the extrapolation length.

+ It is interesting to note that there is a buckling dependence in the constant cross section approximation too. It has been obtained by Sjöstrand [5] in P_3 approximation. It is, however, far below that obtained taking into account the energy dependence.

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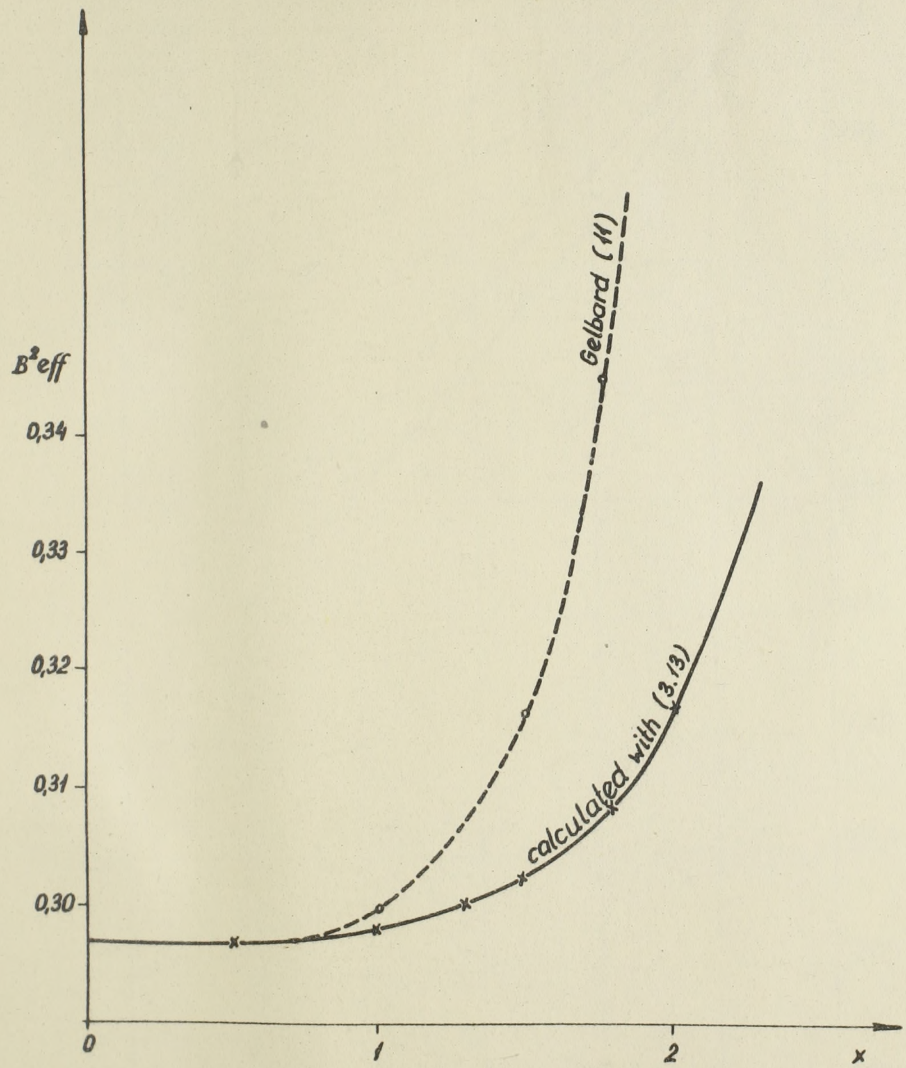

$B^2 eff$
0,34
0,33
0,32
0,31
0,30
0
1
2
x
Gelbard (11)
calculated with (3.13)

Fig. 1.

61751

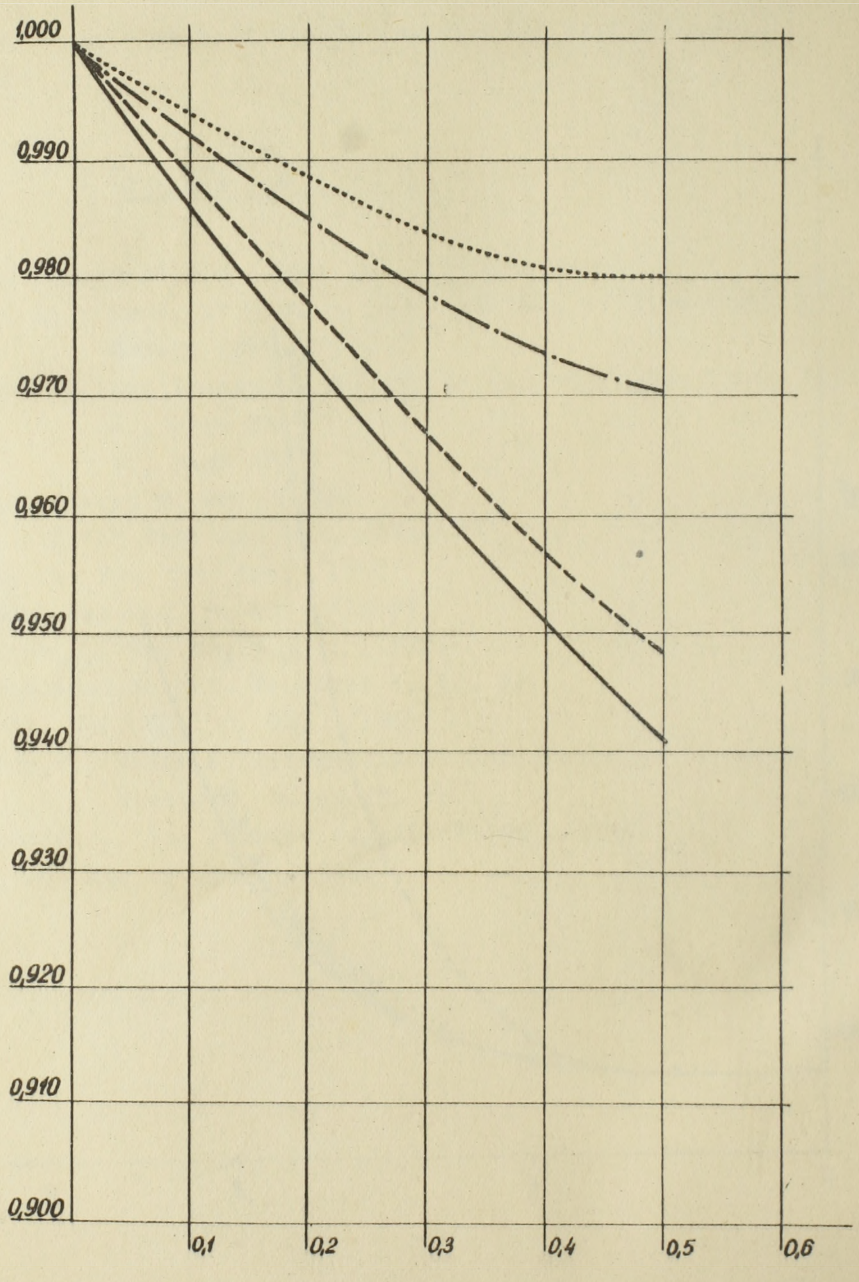

Fig. 2.

—— $\frac{d}{d_0}$ on the base of (4,4)

—·— $\frac{d\,\lambda_{tr}(0)}{d_0\,\lambda_{tr}(B^2)}$ on the base of (4,4)

---- $\frac{d}{d_0}$ Gelbard (11)

.... $\frac{d\,\lambda_{tr}(0)}{d_0\,\lambda_{tr}(B^2)}$ Gelbard (11)